



Early Journal Content on JSTOR, Free to Anyone in the World

This article is one of nearly 500,000 scholarly works digitized and made freely available to everyone in the world by JSTOR.

Known as the Early Journal Content, this set of works include research articles, news, letters, and other writings published in more than 200 of the oldest leading academic journals. The works date from the mid-seventeenth to the early twentieth centuries.

We encourage people to read and share the Early Journal Content openly and to tell others that this resource exists. People may post this content online or redistribute in any way for non-commercial purposes.

Read more about Early Journal Content at <http://about.jstor.org/participate-jstor/individuals/early-journal-content>.

JSTOR is a digital library of academic journals, books, and primary source objects. JSTOR helps people discover, use, and build upon a wide range of content through a powerful research and teaching platform, and preserves this content for future generations. JSTOR is part of ITHAKA, a not-for-profit organization that also includes Ithaka S+R and Portico. For more information about JSTOR, please contact support@jstor.org.

ON THE RATIO OF THE AREA OF A GIVEN TRIANGLE TO THAT OF AN INSCRIBED TRIANGLE.

BY PROF. J. SCHEFFER, HARRISBURGH, PA.

LET us represent the sides BC , AC , AB , respectively by a , b , c ; CD by $a\alpha$, AE by $b\beta$, BF by $c\gamma$; the area of the triangle ABC by Δ and that of the inscribed triangle DEF by Δ' .

We easily find triangle $CDE = \alpha(1 - \beta) \cdot \Delta$; triangle $AEF = \beta(1 - \gamma) \cdot \Delta$; triangle $BDF = \gamma(1 - \alpha) \cdot \Delta$. Therefore

$$\begin{aligned} \frac{\Delta'}{\Delta} &= 1 - \alpha(1 - \beta) - \beta(1 - \gamma) - \gamma(1 - \alpha) \\ &= 1 - (\alpha + \beta + \gamma) + (\alpha\beta + \alpha\gamma + \beta\gamma) \quad (\text{I}) \end{aligned}$$

If the three transversals drawn from A , B , C , to the assumed points D , E , F , intersect in one point we have the relation

$$\alpha\beta\gamma = (1 - \alpha)(1 - \beta)(1 - \gamma), \text{ whence } 1 - (\alpha + \beta + \gamma) + (\alpha\beta + \alpha\gamma + \beta\gamma) = 2\alpha\beta\gamma.$$

In this case therefore

$$\frac{\Delta'}{\Delta} = 2\alpha\beta\gamma. \quad (\text{II})$$

1. Let the three transversals be the medial lines, then $\alpha = \beta = \gamma = \frac{1}{2}$, and according to (II)

$$\frac{\Delta'}{\Delta} = \frac{1}{4}.$$

2. Let the transversals be the bisectors of the angles. From $CD : BD = b : c$, we get

$$CD = \frac{ab}{b+c}, \therefore \alpha = \frac{b}{b+c}, \text{ and similarly } \beta = \frac{c}{a+c}, \gamma = \frac{a}{a+b};$$

$$\therefore \frac{\Delta'}{\Delta} = \frac{2abc}{(a+b)(a+c)(b+c)}, \text{ by (II).}$$

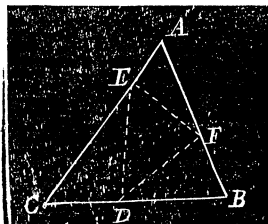
3. Let the transversals be the altitudes

$$CD = b \cos C, AE = c \cos A, BF = a \cos B.$$

$$\therefore \alpha = \frac{b}{a} \cos C, \beta = \frac{c}{b} \cos A, \gamma = \frac{a}{c} \cos B;$$

$$\therefore \frac{\Delta'}{\Delta} = 2 \cos A \cos B \cos C = \frac{(a^2 + b^2 - c^2)(a^2 + c^2 - b^2)(b^2 + c^2 - a^2)}{4a^2b^2c^2}.$$

4. Let the transversals intersect each other in one point and make equal angles with the radii. (Vide Prob. 245, No. 1, Vol. VI.)



$$\begin{aligned} CD &= \frac{ab^2}{b^2+c^2}, \quad AE = \frac{bc^2}{a^2+c^2}, \quad BF = \frac{a^2c}{a^2+b^2}; \\ \therefore a &= \frac{b^2}{b^2+c^2}, \quad \beta = \frac{c^2}{a^2+c^2}, \quad \gamma = \frac{a^2}{a^2+b^2}; \\ \therefore \frac{A'}{A} &= \frac{2a^2b^2c^2}{(a^2+b^2)(a^2+c^2)(b^2+c^2)}. \end{aligned}$$

Compare this result with that in 2.

5. Let the points D, E, F , be the feet of the perpendiculars let fall from the centre of the inscribed circle.

Denote the radius of the inscribed circle by ρ and put $\frac{1}{2}(a+b+c) = s$; then $a = (\rho \div a) \cot \frac{1}{2}C$, $\beta = (\rho \div b) \cot \frac{1}{2}A$, $\gamma = (\rho \div c) \cot \frac{1}{2}B$. Therefore

$$\begin{aligned} \frac{A'}{A} &= \frac{2\rho^3}{abc} \cot \frac{1}{2}A \cot \frac{1}{2}B \cot \frac{1}{2}C = \frac{\rho^2 s}{abc} = \frac{2(s-a)(s-b)(s-c)}{abc} \\ &= \frac{(a+b-c)(a+c-b)(b+c-a)}{4abc}. \end{aligned}$$

Compare this result with that in 3.

FIVE GEOMETRICAL PROPOSITIONS.

BY PROF. ELIAS SCHNEIDER, MILTON, PA.

I. LET A, B, C, D , &c., be the angular points of a regular polygon of n sides, and let AB , one of the equal sides, aqual unity; then will AB be contained once in AC , the chord which contains *two* of the equal sides, with a remainder which call x . Then is $\sqrt{1-x}$ = one side of a polygon of $2n$ sides inscribed in a circle whose radius is *one*.

II. AB will be contained twice in AD , the chord which contains *three* of the equal sides, with a remainder which call y . Then is $\sqrt{1-y}$ = one side of a polygon of n sides inscribed in a circle whose radius is *one*.

III. If the polygon be a Nonagon, AB will be contained twice in AE , the chord which contains *four* of the equal sides, with a remainder which call x . Then is $\sqrt{1-x}$ = one side of a polygon of 18 sides inscribed in a circle whose radius is *one*.

IV. If in Prop. II the polygon be also a Nonagon, then is

$$\sqrt{1-x} = x - y.$$

V. If the polygon be a Decagon, AB will be contained twice in AD , the chord which contains three of the equal sides, with a remainder which call z . Then is $\sqrt{1-z} = z$ = one side of a decagon inscribed in a circle whose radius is *one*.